Reg. No. :

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M.C.A. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

First Semester

MA 5161 – MATHEMATICAL FOUNDATIONS FOR COMPUTER APPLICATIONS

(Regulations 2017)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. If 8, 2 are the two eigenvalues of the square matrix A of order 3 × 3 with determinant value 32, then find the third eigenvalue of A.

2. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

3. How many bit strings of length ten contain exactly four 1's ?

4. Let $R = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 1,3\rangle, \langle 2,2\rangle, \langle 3,2\rangle\}$ be a relation defined on $X = \{1,2,3\}$. Find the matrix of the relation R.

5. Define the conditional statement $P \rightarrow Q$.

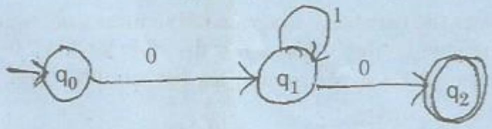
6. Consider the statement "Given any positive integer, there is a greater positive integer". Symbolize this statement with the set of all positive integers as the universe of discourse.

7. Define the phrase structure grammar.

8. Give an example of a language which is not regular but a context free language.

9. Define nondeterministic finite state automata.

10. Find whether the word 100 is accepted by the following finite state automata.



PART – B (5×13=65 Marks)

11\. a) For what values of λ and μ, the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have i) no solution ii) unique solution iii) infinite number of solutions. (13)

(OR)

b) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Hence find the eigenvalues of the matrices A^{-1} and A^4. (13)

12\. a) i) Find the number of integers n, where $1 \le n \le 250$, that are not divisible by any of the integers 2, 3, 5 and 7. (8)

ii) How many solutions does $x + y + z = 20$ have, where x, y and z are non negative integers with $x \ge -1$, $y \ge 0$ and $z \ge 4$? (5)

(OR)

b) i) Let $f : R \to R$ and $g : R \to R$ be any two functions, where R is the set of real numbers. If $f(x) = x^2 - 2$ and $g(x) = x + 4$. Find whether these functions are injective and surjective. (8)

ii) Show that $\overline{P \cup Q} = \overline{P} \cap \overline{Q}$. (5)

13\. a) i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \to R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

ii) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (5)

(OR)

b) i) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (8)

ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (5)

14\. a) i) Consider the grammar G = ({S, A}, {a, b, c}, P, S), where P consists of $S \to A$, $A \to ab$, $A \to aAb$ and $S \to Sc$. Find the language generated by the grammar G. (8)

ii) Classify the types of the grammar. (5)

(OR)

b) State Pumping lemma for regular language. Hence by using it, show that the language $L = \{0^n 1^n$, where, $n \ge 1\}$ is not regular. (13)

15\. a) i) Construct the transition diagram of the finite state automaton to generate the language $L = \{ab^n$, where n is divisible by 3, $n \ge 0\}$. (8)

ii) Construct a NFA to the regular expression $(00 + 11)^* 10$. (5)

(OR)

b) Construct the Deterministic Finite Automaton (DFA) equivalent to the given nondeterministic finite automaton $M = \{\{q_0, q_1\}, \{a, b\}, \delta, q_0, \{q_1\}\}$, where δ is defined as **(13)**

δ	a	b
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

PART – C **(1×15=15 Marks)**

16. a) i) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. Hence find A^{-1}. **(7)**

ii) Use rules of inferences to obtain the conclusion of the following arguments: "Babu, a student in this class, knows how to write programmes in JAVA". "Everyone who knows how to write programmes in JAVA can get a high-paying job". Therefore, "Someone in this class can get a high-paying job". **(8)**

(OR)

b) i) Find the transitive closure of $R = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 3\rangle, \langle 3, 4\rangle, \langle 4, 2\rangle\}$ on the set $A = \{1, 2, 3, 4\}$. **(7)**

ii) Show that the following statements are inconsistent. **(8)**

* If Banu misses many classes due to illness, then he fails in high school.
* If Banu fails in high school, then he is uneducated
* If Banu reads a lot of books, then he is not uneducated
* Banu misses many classes due to illness and reads a lot of books.