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**Question Paper Code : 90338**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Civil Engineering

MA 8353 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to Aeronautical Engineering/Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Marine Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical and Electrochemical Engineering/Food Technology/Pharmaceutical Technology)  
(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Find the complete solution of  $p = 2qx$ .
2. Solve  $(D^2 - 6DD' + 9D'^2)z = 0$ .
3. State the Dirichlet's conditions.
4. Sketch the even extension of the function  $f(x) = \sin x$ ,  $0 < x < \pi$ .
5. Classify the two-dimensional steady state heat conduction equation.
6. Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length  $l$  with insulated ends and with initial temperature  $f(x)$ .
7. State the convolution theorem for Fourier Transforms.

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8. Show that  $\mathfrak{F}_c[f(x)\cos ax] = \frac{1}{2}\{F_c(s+a) + F_c(s-a)\}$  where  $\mathfrak{F}_c[f(x)] = F_c(s)$  is the Fourier cosine transform of  $f(x)$ .
9. Show that  $Z[a^n f(n)] = F\left(\frac{z}{a}\right)$  where  $Z[f(n)] = F(z)$  is the Z-transform of  $f(x)$ .
10. State the initial and final value theorems of Z-transforms.

PART - B

(5×16=80 Marks)

11. a) i) Solve  $(D^3 - 2D^2 D') z = \sin(x + 2y) + 3x^2 y$ . (10)

ii) Form the partial differential equation by eliminating the arbitrary functions from  $u = f(x + ct) + g(x - ct)$ . (6)

(OR)

b) i) Solve  $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$ . (10)

ii) Solve  $p - x^2 = q + y^2$ . (6)

12. a) i) Find the Fourier series of  $f(x) = x^2$  in  $(0, 2l)$ . Hence deduce that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

(10)

ii) Find the complex form of the Fourier series of  $f(x) = \cos ax$  in  $(-\pi, \pi)$ , where 'a' is neither zero nor an integer. (6)

(OR)

b) i) Obtain the constant term and the first three harmonics in the Fourier Cosine series of  $y = f(x)$  in  $(0, 6)$  from the following table. (10)

|   |   |   |    |   |   |   |
|---|---|---|----|---|---|---|
| x | 0 | 1 | 2  | 3 | 4 | 5 |
| y | 4 | 8 | 15 | 7 | 6 | 2 |

ii) Find the Fourier series expansion of  $f(x) = \sin ax$  in  $(-l, l)$ . (6)

13. a) i) Solve  $u_t = a^2 u_{xx}$  by the method of separation of variables and obtain all possible solutions. (8)

ii) A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered as an infinite plate.

If the temperature along the short edge  $y = 0$  is  $u(x, 0) = 100 \sin\left(\frac{\pi x}{8}\right)$ ,  $0 < x < 8$  while two long edges  $x = 0$  &  $x = 8$  as well as the other short edge are kept at  $0^\circ\text{C}$ , then find the steady state temperature at any point of the plate. (8)

(OR)



- b) i) Solve the problem of a tightly stretched string with fixed end points  $x = 0$  &  $x = 1$  which is initially in the position  $y = f(x)$  and which is initially set vibrating by giving to each of its points a velocity  $\frac{dy}{dt} = g(x)$  at  $t = 0$ . (10)
- ii) Classify the partial differential equation  $(1 - x^2) f_{xx} - 2xyf_{xy} + (1 - y^2) f_{yy} = 0$ . (6)
14. a) i) Find the Fourier transform of  $f(x)$  where  $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$  and hence evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$ . (10)
- ii) Show that  $\frac{1}{\sqrt{x}}$  is self-reciprocal under the Fourier cosine transform. (6)  
(OR)
- b) i) Find the Fourier cosine and sine transforms of  $e^{-ax}$ ,  $a > 0$  and hence deduce their inversion formulae. (10)
- ii) Using Parseval's identity, evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$ ,  $a > 0$ . (6)
15. a) i) Find  $Z\{\sin bt\}$  and hence find  $Z\{e^{-at} \sin bt\}$ . (8)
- ii) Find  $Z^{-1}\left\{\frac{8z^2}{(2z-1)(4z+1)}\right\}$  using convolution theorem. (8)  
(OR)
- b) i) Using Z-transforms, solve the difference equation  $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$  given  $y_0 = y_1 = 0$ . Use partial fraction method to find the inverse Z-transform. (8)
- ii) Using residue method, find  $Z^{-1}\left\{\frac{z}{z^2 + 2z + 2}\right\}$ . (8)