Reg. No. :

Question Paper Code : 31521

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Second Semester

Civil Engineering

MA 2161/MA 22/080030004 — MATHEMATICS — II

(Common to all branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Solve $(D^2-4)y=1$.
2. Convert $(3x^2D^2+5xD+7)y=2/x\log x$ into an equation with constant coefficients.
3. Define solenoidal vector function. If $\vec{V}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+2\lambda z)\vec{k}$ is solenoidal, find the value of λ.
4. State Green's theorem.
5. Find the constants a, b if $f(z)=x+2ay+i(3x+by)$ is analytic.
6. Find the critical points of the transformation $w=1+\dfrac{2}{z}$.
7. Evaluate $\int_C \dfrac{z+4}{z^2+2z}$ where C is the circle $\left|z-\dfrac{1}{2}\right|=\dfrac{1}{3}$.
8. Find the residue of $f(z)=\dfrac{1-e^{-z}}{z^3}$ at $z=0$.
9. Find the Laplace transform of $f(t)=\dfrac{1-e^{-t}}{t}$.
10. Find the Laplace transform of the function $f(t)=\begin{cases}1, & t=0\\ 0, & t\neq 0\end{cases}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=8xe^x\sin x$. (8)

(ii) Solve by the method of variation of parameters $2\dfrac{d^2y}{dx^2}+8y=\tan 2x$. (8)

Or

(b) (i) Solve $x^2\dfrac{d^2y}{dx^2}+4x\dfrac{dy}{dx}+y=e^{e^{\log x}}$. (8)

(ii) Solve $\dfrac{dx}{dt}+4x+3y=t$; $\dfrac{dy}{dt}+2x+5y=e^{2t}$. (8)

12. (a) (i) Show that the vector field $\overline{F} = (x^2 + xy^2)\overline{i} + (y^2 + x^2y)\overline{j}$ is irrotational. Find its scalar potential. (6)

(ii) Verify Stoke's theorem for $\overline{F} = (x^2 + y^2)\overline{i} - 2xy\overline{j}$ taken around the rectangle formed by the lines $x = -a$, $x = +a$, $y = 0$ and $y = b$. (10)

Or

(b) (i) Find a and b so that the surfaces $ax^3 - by^2z - (a+3)x^2 = 0$ and $4x^2y - z^3 - 11 = 0$ cut orthogonally at the point $(2, -1, -3)$. (6)

(ii) Verify Gauss Divergence theorem for $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$, where S is the surface of the cube formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (10)

13. (a) (i) Prove that $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the corresponding analytic function and the imaginary part. (8)

(ii) Find the bilinear map which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also find the image of the unit circle of the z plane. (8)

Or

(b) (i) Prove that $w = \dfrac{z}{1-z}$ maps the upper half of the z-plane to the upper half of the w-plane and also find the image of the unit circle of the z plane. (8)

(ii) Find the analytic function $f(z) = u + iv$ where $v = 3r^2\sin 2\theta - 2r\sin\theta$. Verify that u is a harmonic function. (8)

14. (a) (i) Find the residues of $f(z) = \dfrac{z^2}{(z+2)(z-1)^2}$ at its isolated singularities using Laurentz's series expansion. (8)

(ii) Evaluate $\displaystyle\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\,d\theta$, using contour integration. (8)

Or

(b) (i) Show that $\displaystyle\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\,dx = \frac{5\pi}{2}$. (8)

(ii) Evaluate $\displaystyle\int_C \frac{z+1}{(z^2 + 2z + 4)^2}\,dz$, where C is the circle $|z + 1 + i| = 2$, by Cauchy's integral formula. (8)

15. (a) (i) Evaluate $L^{-1}\left(\dfrac{3s^2 + 16s + 26}{s(s^2 + 4s + 13)}\right)$. (8)

(ii) Find the inverse Laplace transform of the following : $\log\left(\dfrac{s+1}{s-1}\right)$. (8)

Or

(b) (i) Find $L^{-1}\left[\dfrac{s}{(s^2 + a^2)^2}\right]$ and find $L^{-1}\left[\dfrac{1}{(s^2 + a^2)^2}\right]$ and hence find $L^{-1}\left(\dfrac{1}{(s^2 + 9s + 13)^2}\right)$. (3 + 3 + 2)

(ii) Using Laplace transforms, solve $y'' + y' = t^2 + 2t$, $y(0) = 4$ and $y'(0) = -2$. (8)
