Reg. No. : [ ]

Question Paper Code : 71769

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to all branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ then find the eigen values of A^{-1}.

2. State Cayley-Hamilton theorem.

3. Find the centre and radius of the sphere $4(x^2 + y^2 + z^2) - 8x + 12y - 16z - 20 = 0$.

4. Find the equation of the right circular cone whose vertex is origin, axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi vertical angle is 30°.

5. Find the radius of curvature of the curve given by $y = c \log \sec \frac{x}{c}$.

6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$ where m is the parameter and a is constant.

7. If $u = \frac{x + y}{xy}$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

8. State Euler's theorem for homogeneous function.

9. Evaluate $\int_0^3 \int_0^2 e^{x+y}\, dy\, dx$.

10. Express volume of a solid as a triple integral.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. (8)

(ii) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and hence find A^{-1}. (8)

Or

(b) Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ into a canonical form by an orthogonal transformation and hence find its nature. (16)

12. (a) (i) Find the centre and radius of the circle given by the equations $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$, $x - 2y + 2z - 3 = 0$. (8)

(ii) Find the equation of the cone whose vertex is origin and guiding curve the circle $x^2 + y^2 + z^2 + 2x - y + 3z - 1 = 0$, $x - y + z + 4 = 0$. (8)

Or

(b) Find the equation of the cylinder whose generators are parallel to the line $x = y = z$ and whose guiding curve is the circle $x^2 + y^2 + z^2 - 2x - 3 = 0$, $2x + y + 2z = 0$. (16)

13. (a) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (16)

Or

(b) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (16)

14. (a) (i) If $u = \tan^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$. (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2\, x_3}{x_1}$, $y_2 = \frac{x_3\, x_1}{x_2}$, $y_3 = \frac{x_1\, x_2}{x_3}$. (8)

Or

(b) (i) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ as a Taylor series about the point (1,1) upto 2nd degree terms. (8)

(ii) Find the shortest distance from the point (1,0) to the parabola $y^2 = 4x$. (8)

15. (a) (i) Evaluate $\iint_R xy\, dx\, dy$, where R is the region bounded by the lines, $x = 0$, $y = 0$ and $x + 2y = 2$. (8)

(ii) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration. (8)

Or

(b) (i) Evaluate $\int_0^1 \int_y^{2-y} xy\, dx\, dy$ by changing the order of integration. (8)

(ii) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\, dy\, dx}{(x+y+z+1)^3}$. (8)
