Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 11483

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2013.

First Semester

(Common to all Branches)

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the symmetric matrix A, whose eigenvalues are 1 and 3 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

2. Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 1 \\ -2 & 1 & -2 \end{bmatrix}$.

3. Find the equation of the sphere whose centre is (1, 2, –1) and which touches the plane $2x - y + z + 3 = 0$.

4. Find the radius of curvature of the curve $x^2 + y^2 - 4x + 2y - 8 = 0$.

5. Find the equation of the right circular cylinder whose axis is z-axis and radius is 'a'.

6. Find the envelope of the lines $x \operatorname{cosec}\theta - y \cot\theta = a$, θ being the parameter.

7. If $u = f(y - z, z - x, x - y)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

8. If $r = \frac{yz}{x}$, $s = \frac{zx}{y}$, $t = \frac{xy}{z}$, find $\frac{\partial(r, s, t)}{\partial(x, y, z)}$.

9. Plot the region of integration to evaluate the integral $\iint_D f(x, y)\, dx\, dy$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

10. Evaluate $\int_0^2 \int_0^{\pi} r \sin^2 \theta \, d\theta dr$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$. (8)

(ii) If the eigenvalues of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 0, 3, 15, find the eigenvectors of A and diagonalize the matrix A. (8)

Or

(b) (i) Reduce the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$ into canonical form. (8)

(ii) Show that the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ satisfies its own characteristic equation. Find also its inverse. (8)

12. (a) (i) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y + 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. Find also their points of contact. (8)

(ii) Find the equation of the right circular cone whose vertex is (2, 1, 0), semivertical angle is 30° and the axis is the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z}{2}$. (8)

Or

(b) (i) Find the equation of the cylinder whose generators are parallel to $\frac{x}{2} = \frac{y}{2} = \frac{z}{-3}$ and whose guiding curve is the ellipse $3x^2 + y^3 = 3$, $z = 2$.

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and also find the point of contact. (8)

13. (a) (i) Find the envelope of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters are related by the equation $a^2 + b^2 = c^2$. (8)

(ii) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$. (8)

Or

(b) (i) Find the radius of curvature and centre of curvature of the parabola $y^2 = 4ax$ at the point t. Also find the equation of the evolute. (10)

(ii) Find the envelope of the circles drawn upon the radius vectors of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter. (6)

14. (a) (i) If $u = e^{xy}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$. (8)

(ii) Test for the maxima and minima of the function $f(x, y) = x^3 y^2 (6 - x - y)$. (8)

Or

(b) (i) If F is a function of x and y and if $x = e^u \sin v$, $y = e^u \cos v$, prove that $\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = e^{-2u}\left[\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial v^2}\right]$. (8)

(ii) If $x^2 + y^2 + z^2 = r^2$, show that the maximum value of $yz + zx + xy$ is r^2 and the minimum value is $-\frac{r^2}{2}$. (8)

15. (a) (i) Change the order of integration in the integral $\int_0^a \int_{x^2/2}^{2a-x} xy\,dx\,dy$ and evaluate it. (8)

(ii) Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ for all positive values of x, y, z for which the integral is real. (8)

Or

(b) (i) By transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\,dx\,dy$ over the annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $(b>a)$. (8)

(ii) Find the area which is inside the circle $r = 3a\cos\theta$ and outside the cardioid $r = a(1+\cos\theta)$. (8)
