Reg. No. : [ ]

Question Paper Code : 31519

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to All Branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue.

2. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

3. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 6x - 6y + 8z + 9 = 0$.

4. Prove that the equation $x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$ represents a cone with vertex $(1, -2, 3)$.

5. Find the radius of curvature of the curve $xy = c^2$ at (c, c).

6. Find the envelope of the lines $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$, θ being the parameter.

7. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

8. Find the Taylor series expansion of x^y near the point $(1,1)$ upto the first degree terms.

9. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

10. Change the order of integration in $\int_0^{1}\int_0^{2\sqrt{x}} f(x,y)\,dy\,dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem find A^{-1} for the matrix $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $Q = 3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz$ to its canonical form using orthogonal transformation. Also find its rank, index and signature. (16)

12. (a) (i) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 + 2x - 2y + 4z - 19 = 0$ and $x + 2y + 2z + 7 = 0$. (8)

(ii) Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose base in the curve $y = 0$, $x^2 + z^2 = 4$. (8)

Or

(b) (i) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. (8)

(ii) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$. (8)

13. (a) (i) Prove that for the curve $y = \frac{ax}{a+x}$, $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. (8)

(ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (8)

Or

(b) (i) Obtain the equation of the evolute of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$. (8)

(ii) Prove that the radius of curvature of the curve $xy^2 = a^3 - x^3$ at the point $(a, 0)$ is $\frac{3a}{2}$. (8)

14. (a) (i) If $z = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$ prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. (8)

(ii) Discuss the maxima and minima of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$. (8)

Or

(b) (i) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ prove that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$. (6)

(ii) A rectangular box open at the top is to have a capacity of 108 cu.ms. Find the dimensions of the box requiring the least material for its construction. (10)

15. (a) (i) Evaluate $\iint xy\,dx\,dy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$. (6)

(ii) Find the value of $\iiint xyz\,dx\,dy\,dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \le a^2$. (10)

Or

(b) (i) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dy\,dx$ and hence evaluate it. (8)

(ii) Evaluate, by changing to polar co-ordinates, the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}\,dx\,dy$. (8)
