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Question Paper Code : 71774

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/080250008/10177 PQ 401 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

Use of statistical tables may be permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ a & ; \quad 1 \le x \le 2 \\ 3a - ax & ; \quad 2 \le x \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

then find the value of 'a'.

2. Suppose that, on an average, in every three pages of a book there is one typographical error. If the number of typographical errors on a single page of the book is a Poisson random variable. What is the probability of at least one error on a specific page of the book?

3. The joint probability mass function of a two dimensional random variable (X,Y) is given by $p(x, y) = k(2x + 3y)$; $x = 0,1,2$; $y = 1, 2, 3$. Find the value of k.

4. What do you mean by correlation between two random variables?

5. What is a random process? When do we say a random process is a random variable?

6. Is Poisson process stationary? Justify.

7. Draw the state transition rate diagram of a M/M/C queueing model.

8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queueing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

9. State Jackson's theorem for an open network.

10. What do the letter in the symbolic representation $M/G/1$ of a queueing model represent?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A random variable X has the following probability distribution :

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find

(1) the value of k (8)

(2) $P(1.5 < X < 4.5 / X > 2)$

(ii) Find the MGF of the binomial distribution and hence find its mean and variance. (8)

Or

(b) (i) The distribution function of a random variable X is given by $F(x) = 1-(1+x)e^{-x}; x \geq 0$. Find the density function, mean and variance of X. (8)

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tires will last at least 20,000 km and at most 30,000 km. (8)

12. (a) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$; $0 \le x \le 2$; $0 \le y \le 1$. Compute $P(X > 1)$, $P\left(Y < \frac{1}{2}\right)$, $P\left(X > 1/Y < \frac{1}{2}\right)$, $P\left(Y < \frac{1}{2}/X > 1\right)$; $P(X < Y)$ and $P(X + Y \le 1)$. (16)

Or

(b) Obtain the equations of the regression lines from the following data. Hence find the coefficient of correlation between X and Y. Also estimate the value of Y when $X = 38$ and X when $Y = 18$. (16)

X:	22	26	29	30	31	31	34	35
Y:	20	20	21	29	27	24	27	31

13. (a) (i) A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Is $\{X_n;\ n = 1, 2, \ldots\}$ a Markov chain? If so, find its transition probability matrix, also specify the classes. (8)

(ii) Show that the process $X(t) = A \cos \lambda t + B \sin \lambda t$ where A and B are random variables, is wide-sense stationary, if $E(A) = E(B) = 0$ and $E(A^2) = E(B^2)$; $E(AB) = 0$. (8)

Or

(b) (i) Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute; find the probability that during a time interval of 2 mins (1) exactly 4 customers arrive and (2) more than 4 customers arrive. (8)

(ii) An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout? (8)

14. (a) Customers arrive at a one-man barber shop according to Poisson process with a mean inter arrival time of 12 mins. Customers spend on average of 10 mins in the barber's chair.

(i) What is the expected number of customers in the barber shop and in the queue?

(ii) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.

(iii) How much time can a customer expect to spend in the barber's shop?

(iv) What is the probability that the waiting time in the system is greater than 30 mins? (16)

Or

(b) A tax-consulting firm has 3 counters in its office to receive people who have problems concerning their income, wealth and sales taxes. On the averages 48 persons arrive in an 8-hr day. Each tax advisor spends 15 mins on the average on an arrival. If the arrivals are Poisson distributed and service times are according to exponential distribution, find

(i) the average number of customers in the system

(ii) the average number of customers waiting to be serviced

(iii) the average time a customer spends in the system. (16)

15. (a) Derive Pollaczek-Khintchine formula of an $M/G/1$ queueing model. (16)

Or

(b) (i) Write a brief note on the open queueing networks. (8)

(ii) A repair facility shared by a large number of machines has 2 series stations with respective service rates of 2 per hour and 3 per hour. If the average rate of arrival is 1 per hour, find

(1) the average number of machines in the system.

(2) the average waiting time in the system

(3) probability that both service stations are idle. (8)