CSE - IV sem

Reg. No. :

Question Paper Code : 21524

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/10177 PQ 401/080250008 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008/2010)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If X and Y are two independent random variables with variances 2 and 3, find the variance of $3X + 4Y$.

2. State memory less property of exponential distribution.

3. If the joint pdf of (X,Y) is given by $f(x,y) = 2$, in $0 \le x < y \le 1$, find $E(X)$.

4. State Central limit theorem.

5. Define wide sense stationary process.

6. If the transition probability matrix (tpm) of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the steady state distribution of the chain.

7. What are the characteristics of a queuing system?

8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queuing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

9. State Pollaczek-Khinchine formula.

10. Define closed network of a queuing system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A continuous random variable has the pdf $f(x) = kx^4, -1 < x < 0$. Find the value of k and also $P\left\{X > \left(-\frac{1}{2}\right) \Big/ X < \left(-\frac{1}{4}\right)\right\}$. (8)

(ii) Find the moment generating function of Uniform distribution. Hence find its mean and variance. (8)

Or

(b) (i) Find the moment generating function and r^th^ moment for the distribution whose pdf is $f(x) = Ke^{-x}, 0 \le x \le \infty$. Hence find the mean and variance. (8)

(ii) In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) at most there are 3 defective bulbs (3) exactly there are 3 defective bulbs. (8)

12. (a) (i) The joint probability density function of a two-dimensional random variable (X,Y) is $f(x,y) = \frac{1}{8}(6 - x - y), 0 < x < 2, 2 < y < 4$. Find (1) $P(X < 1 \cap Y < 3)$ (2) $P(X + Y < 3)$ (3) $P(X < 1/Y < 3)$. (8)

(ii) If X and Y each follow an exponential distribution with parameter I and are independent, find the pdf of $U = X - Y$. (8)

Or

(b) (i) The marks obtained by 10 students in Mathematics and Statistics are given below. Find the correlation coefficient between the two subjects. (8)

Marks in mathematics	75	30	60	80	53	35	15	40	38	48
Marks in statistics	85	45	54	91	58	63	35	43	45	44

(ii) A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (8)

13. (a) (i) Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ is wide sense stationary, if $E(A) = E(B) = 0, E(A^2) = E(B^2)$ and $E(AB) = 0$, where A and B are random variables. (8)

(ii) A gambler has Rs 2. He bets Re. 1 at a time and wins Re. 1 with probability $\frac{1}{2}$. He stops playing if he loses Rs. 2 or wins Rs. 4. (1) What is the tpm of the related Markov chain? (2) What is the probability that he has lost his money at the end of 5 plays? (8)

Or

(b) (i) Find the nature of the states of the Markov chain with the tpm $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$. (8)

(ii) Prove that the difference of two independent Poisson processes is not a Poisson process. (4)

(iii) Prove that the Poisson process is a Markov Process. (4)

14. (a) (i) Derive (1) L_s, average number of customers in the system (2) L_q, average number of customers in the queue for the queuing model (M/M/1): (N/FIFO). (8)

(ii) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy? What is the average number of letters waiting to be typed? (Assume Poisson arrivals and exponential service times) (8)

Or

(b) Customers arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then

(i) What is the probability that a customer need not wait for a hair cut?

(ii) What is the expected number of customer in the barber shop and in the queue?

(iii) How much time can a customer expect to spend in the barber shop?

(iv) Find the average time that a customer spend in the queue.

(v) Estimate the fraction of the day that the customer will be idle?

(vi) What is the probability that there will be 6 or more customers?

(vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair. (16)

15. (a) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process at the rate of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The service time for all cars is constant and equal to 10 minutes. Determine L_s, L_q, W_s and W_q. (16)

Or

(b) Consider a system of two servers where customers from outside the system arrive at sever 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. (16)
