CSE - IV Sem

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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fourth Semester

Computer Science and Engineering

MA 2262/MA 44/MA 1252/10177 PQ 401/080250008 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

(Use of statistical tables may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A coin is tossed 2 times, if 'X' denotes the number of heads, find the probability distribution of X.

2. If the probability that a target is destroyed on any one shot is 0.5, find the probability that it would be destroyed on 6^{th} attempt.

3. If the joint pdf of the two-dimensional random variable (X, Y) is given by $f(x, y) = K_{xy}\, e^{-(x^2+y^2)}$, $x > 0$, $y > 0$ find the value of K.

4. State central limit theorem.

5. Prove that first order stationary random process has a constant mean.

6. Prove that Poisson process is a Markov process.

7. Define Markovian Queueing Models.

8. Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes.
   (a) What is the average number of customers in the system?
   (b) What is the average time of a customer spends in the system?

9. Define series queues.

10. Define open network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the MGF of the random variable 'X' having the pdf (8)

$$f(x) = \begin{cases} x, & for \quad 0 < x < 1 \\ 2 - x, & for \quad 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(ii) A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the probability that a box fail to meet the guaranteed quality? (8)

Or

(b) (i) 6 dice are thrown 729 times. How many times do you expect atleast three dice to show a five (or) a six? (8)

(ii) If a continuous RV, X follows uniform distribution in the interval (0, 2) and a continuous RV, Y follows exponential distribution with parameter λ. Find λ such that $P(X < 1) = P(Y < 1)$. (8)

12. (a) (i) Let X and Y be random variables having joint density function

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the correlation co-efficient γ_{xy}. (8)

(ii) The joint distribution of X and Y is given by $f(x, y) = \frac{x + y}{21}$, $x = 1, 2, 3,\ y = 1, 2$. Find the marginal distributions. (8)

Or

(b) (i) If the pdf of 'X' is $f_X(x) = 2x,\ 0 < x < 1$, find the pdf of $Y = 3X + 1$. (8)

(ii) The life time of a certain band of an electric bulb may be considered as a RV with mean 1200 h and SD 250 h. Using central limit theorem, find the probability that the average life time of 60 bulbs exceeds 1250 h. (8)

13. (a) (i) Prove that the process $\{X(t)\}$ whose probability distribution given by $P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, ... \\ \frac{at}{1+at}, & n = 0 \end{cases}$ is not stationary. (8)

(ii) The TPM of a Markov chain $\{X_n\}$, $n = 1, 2, 3, \ldots$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = \{0.7,\ 0.2,\ 0.1\}$ find

(1) $P[X_2 = 3]$

(2) $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (8)

Or

(b) (i) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However if he sells in either B or C the next day he is twice as likely as to sell in city A as in the other city. In the long run how often does he sell in each the cities? (8)

(ii) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes

(1) Exactly 4 customers arrive

(2) More than 4 customers arrive

(3) Less than 4 customers arrive. (8)

14. (a) (i) A T.V. repairman finds that the time spend on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of average set just brought? (8)

(ii) Consider a single server queueing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is two. Find the steady state probability distribution of the number of calling units in the system and the expected number of calling units in the system. (8)

Or

(b) (i) A telephone exchange has two long distance operators. It is observed that, during the peak load long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.

Find :

(1) The probability a subscriber will have to wait for long distance call during the peak hours of the day.

(2) If the subscribers will wait and are serviced in turn, what is the expected waiting time? (8)

(ii) Customers arrive at a sales counter manned by a single person according to a Poisson process with mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer. (8)

15. (a) (i) A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with mean of 4 cars per hour and may wait in the factory's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for a car has uniform distribution between 8 and 12 minutes Find :

(1) The average number of cars waiting in the parking lot and

(2) The average waiting time of a car in the parking lot. (8)

(ii) There are two salesmen in a ration shop one incharge of billing and receiving payment and the other incharge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop, that too when the billing clerk is free. The customer who has finished his billing job has to wait there until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rate of 3 and 2 find

(1) The proportion of customers who enter the ration shop.

(2) The average number of customers in the shop

(3) The average amount of time that an entering customer spends in the shop. (8)

Or

(b) (i) Derive Pollaczek – Khintchine formula. (8)

(ii) A one man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. (8)

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