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Question Paper Code : 11490

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Fifth Semester

Computer Science and Engineering

MA 2265 — DISCRETE MATHEMATICS

(Common to Information Technology)

(Common to PTME 2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulation 2009)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Define Tautology with an example.

2. Define a rule of Universal specification.

3. State Pigeonhole principle.

4. Solve : $a_k = 3a_{k-1}$, for $k \geq 1$, with $a_0 = 2$.

5. Define a regular graph. Can a complete graph be a regular graph?

6. State the handshaking theorem.

7. Prove that the identity of a subgroup is the same as that of the group.

8. State Lagrange's theorem in δ group theory.

9. When is a lattice said to be bounded?

10. When is a lattice said to be a Boolean Algebra?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (8)

(ii) Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". (8)

Or

(b) (i) Find the principal disjunctive normal form of the statement, $(q \vee (p \wedge r)) \wedge \sim ((p \vee r) \wedge q)$. (8)

(ii) Use the indirect method to prove that the conclusion $\exists z Q(z)$ follows from the premises $\forall x (P(x) \rightarrow Q(x))$ and $\exists y P(y)$. (8)

12. (a) (i) Use mathematical induction to prove the inequality $n < 2^n$ for all positive integer n. (8)

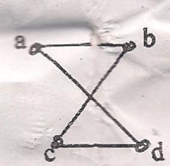
(ii) What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade if there are five possible grades A, B, C, D and F? (8)

Or

(b) (i) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? (8)

(ii) Use generating functions to solve the recurrence relation $a_n + 3a_{n-1} - 4a_{n-2} = 0$, $n \geq 2$ with the initial condition $a_0 = 3$, $a_1 = -2$. (8)

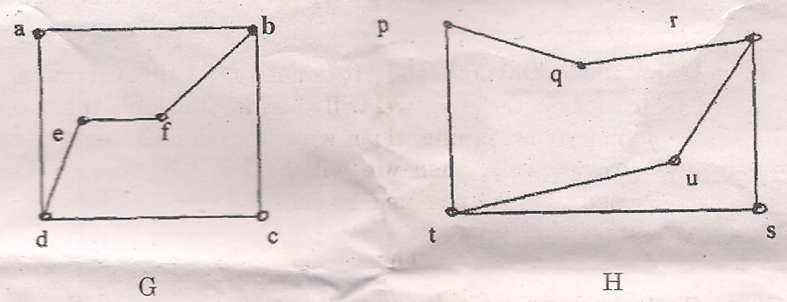
13. (a) (i) How many paths of length four are there from a to d in the simple graph G given below. (8)



(ii) Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. (8)

Or

(b) (i) Determine whether the graphs G and H given below are isomorphic. (8)



(ii) Prove that an undirected graph has an even number of vertices of odd degree. (8)

14. (a) (i) If $*$ is the operation defined on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a,b)*(x,y) = (ax, ay+b)$, show that $(S,*)$ is a semi group. Is it commutative? Also find the identity element of S. (8)

(ii) Prove that the necessary and sufficient condition for a non empty subset H of a group $\{G,*\}$ to be a sub group is $a,b \in H \Rightarrow a*b^{-1} \in H$. (8)

Or

(b) (i) Prove that the set $Z_4 = \{[0],[1],[2],[3]\}$ is a commutative ring with respect to the binary operation addition modulo and multiplication modulo $+_4, \times_4$. (8)

(ii) If $f: G \to G'$ is a group homomorphism from $\{G,*\}$ to $\{G',\Delta\}$ then prove that for any $a \in G$, $f(a^{-1}) = [f(a)]^{-1}$. (8)

15. (a) (i) Draw the Hasse diagram representing the partial ordering $\{(A,B): A \subseteq B\}$ on the power set $P(S)$ where $S = \{a,b,c\}$. Find the maximal, minimal, greatest and least elements of the poset. (8)

(ii) In a Boolean algebra, prove that $a.(a+b) = a$, for all $a,b \in B$. (8)

Or

(b) (i) In a distributive Lattice $\{L, \vee, \wedge\}$ if an element $a \in L$ a complement then it is unique. (8)

(ii) Simplify the Boolean expression $a'.b'.c + a.b'.c + a'.b'.c'$ using Boolean algebra identities. (8)

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