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Question Paper Code : 55443

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2011.

Fifth Semester

Computer Science and Engineering

MA 2265 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Show that the propositions $p \to q$ and $\neg p \vee q$ are logically equivalent.

2. Give an indirect proof of the theorem "If $3n + 2$ is odd, then n is odd".

3. Write the generating function for the sequence $1, a, a^2, a^3, a^4, \ldots$.

4. Use mathematical induction to show that $n! \geq 2^{n+1}$, $n = 1, 2, 3, \ldots$

5. When is a simple graph G bipartite? Give an example.

6. Define complete graph and give an example.

7. Define homomorphism and isomorphism between two algebraic systems.

8. When is a group $(G, *)$ called abelian?

9. Let $A = \{a, b, c\}$ and $\rho(A)$ be its power set. Draw a Hasse diagram of $< \rho(A), \subseteq >$.

10. When is a lattice called complete?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using indirect method of proof, derive $p \to \neg s$ from the premises $p \to (q \vee r)$, $q \to \neg p$, $s \to \neg r$ and p. (8)

(ii) Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction. (8)

Or

(b) (i) Show that $\forall x(P(x) \vee Q(x)) \Rightarrow (\forall x\ P(x)) \vee (\exists x\ Q(x))$ by indirect method of proof. (8)

(ii) Show that the statement "Every positive integer is the sum of the squares of three integers" is false. (8)

12. (a) (i) If n Pigeonholes are occupied by $(kn + 1)$ pigeons, where k is positive integer, prove that at least one Pigeonhole is occupied by
$(k + 1)$ or more Pigeons. Hence, find the minimum number of m integers to be selected from S = {1,2, ..., 9} so that the sum of two
of the m integers are even. (8)

(ii) Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n, n \geq 0$, $a_0 = 3$. (8)

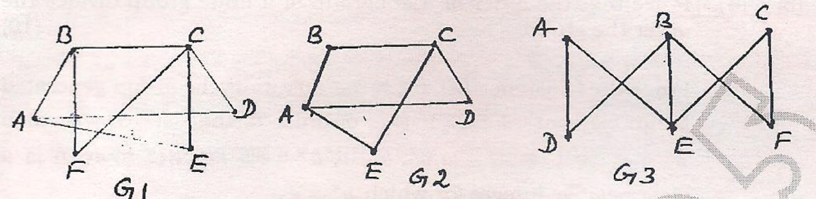
Or

(b) (i) Use mathematical induction to show that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$, $n \geq 2$. (8)

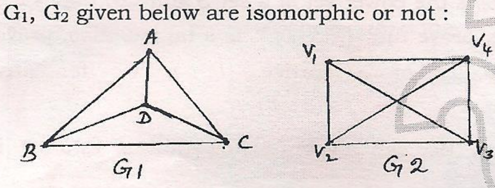
(ii) Use the method of generating function to solve the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2} + 4^n;\ n \geq 2, \text{ given that } a_0 = 2 \text{ and } a_1 = 8.$$ (8)

13. (a) (i) Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite. (8)



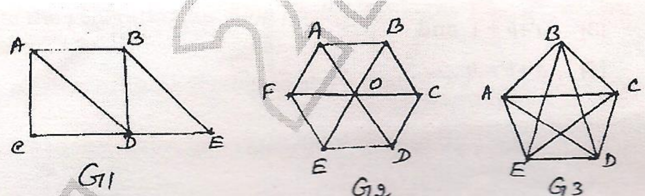
(ii) Using circuits, examine whether the following pairs of graphs G_1, G_2 given below are isomorphic or not : (8)



Or

(b) (i) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (10)

(ii) Find an Euler path or an Euler circuit, if it exists in each of the three graphs below. If it does not exist, explain why? (6)



14. (a) (i) Let $(S, *)$ be a semigroup. Then prove that there exists a homomorphism $g : S \to S^S$, where $\langle S^S, \circ \rangle$ is a semigroup of functions from S to S under the operation of (left) composition. (8)

(ii) Prove that every finite group of order n is isomorphic to a permutation group of order n. (8)

Or

(b) (i) Prove that the order of a subgroup of a finite group divides the order the group. (10)

(ii) Prove the theorem : Let $\langle G, *\rangle$ be a finite cyclic group generated by an element $a \in G$. If G is of order n, that is, $|G| = n$, then $a^n = e$, so that $G = \{a, a^2, a^3, ..., a^n = e\}$. Further more n is a least positive integer for which $a^n = e$. (6)

15. (a) (i) If $P(S)$ is the power set of a set S and $\cup$, $\cap$ are taken as join and meet, prove that $\langle P(S), \subseteq\rangle$ is a lattice. Also, prove the modular inequality of a Lattice $\langle L, \leq\rangle$, viz for any $a, b, c \in L$, $a \leq c \Leftrightarrow a \vee (b \wedge c) \leq (a \vee b) \wedge c$. (10)

(ii) In any Boolean algebra, show that $ab' + a'b = 0$ if and only if $a = b$. (6)

Or

(b) (i) Prove that Demorgan's laws hold good for a complemented distributive lattice $\langle L, \wedge, \vee\rangle$, viz $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (8)

(ii) In any Boolean algebra, prove that the following statements are equivalent : (8)

(1) $a + b = b$

(2) $a \bullet b = a$

(3) $a' + b = 1$ and

(4) $a \bullet b' = 0$.
