

Reg. No. :

**Question Paper Code : 71773**

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND  
RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours

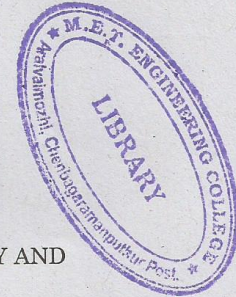
Maximum : 100 marks

(Use of Statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If a random variable  $X$  takes values 1, 2, 3, 4 such that  $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$ . Find the probability distribution of  $X$ .
2. Find the moment generating function of Poisson distribution.
3. The joint pdf of  $(X, Y)$  is given by  $f(x, y) = k xye^{-(x^2+y^2)}$ ;  $x > 0, y > 0$ . Find the value of  $k$ .
4. Define the distribution function of two dimensional random variable  $(X, Y)$ . State any one property.
5. Define a Markov process.
6. Prove that the sum of two independent Poisson processes is a Poisson process.
7. Define power spectral density function of a stationary random process.
8. If  $R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$ . Find the mean and variance of  $X$ .
9. Define a linear system with random output.
10. State any two properties of cross power density spectrum.





PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable  $X$  has cdf  $F(x) = \begin{cases} 0 & \text{if } x < -1 \\ a(1+x) & \text{if } -1 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$ . Find the value of  $a$ . Find  $P(X > \frac{1}{4})$  and  $P(-0.5 \leq X \leq 0)$ . (8)
- (ii) Obtain the moment generating function of geometric distribution. Hence, find its mean and variance. (8)

Or

- (b) (i) If  $X$  is uniformly distributed with  $E(X) = 1$  and  $\text{var}(X) = 4/3$ , find  $P(X < 0)$ . (8)
- (ii) Obtain the moment generating function of exponential distribution. Hence compute the first four moments. (8)
12. (a) (i) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 blue balls. If  $X$  denotes the number of white balls drawn and  $Y$  denotes the number of red balls drawn, find the probability distribution of  $X$  and  $Y$ . (8)
- (ii) The random variables  $X$  and  $Y$  are related by  $X - 6 = Y$  and  $0.64X - 4.08 = 0$ . Find the mean of  $X$  and  $Y$ ; and correlation coefficient between  $X$  and  $Y$ . (8)

Or

- (b) (i) The random variables  $X$  and  $Y$  have joint pdf  $f(x, y) = \begin{cases} x^2 + \frac{xy}{3}; & 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$ . Find the marginal density of  $X$  and marginal density of  $Y$ . Find the conditional density of  $X$  given  $Y$ . (8)
- (ii) A random sample of size 100 is taken from a population whose mean  $\mu = 60$  and variance  $\sigma^2 = 400$ . Using central limit theorem with what probability can we assert that the mean of the sample will not differ from  $\mu$  by more than 4. (8)
13. (a) (i) Examine whether  $X(t) = A \cos \lambda t + B \sin \lambda t$  where  $A$  and  $B$  are random variables such that  $E(A) = E(B) = 0$ ;  $E(A^2) = E(B^2)$ ;  $E(AB) = 0$ , is wide sense stationary. (8)
- (ii) Find the auto correlation function of the Poisson process. (8)

Or



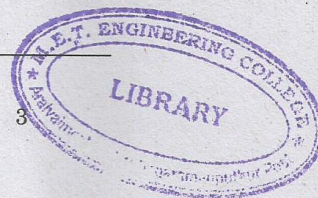
- (b) (i) Suppose  $X(t)$  is a normal process with mean  $\mu(t)=3$ ,  $C_x(t_1, t_2) = 4e^{-0.2|t_1-t_2|}$ . Find  $P(X(5) \leq 2)$  and  $P(|X(8) - X(5)| \leq 1)$ . (8)
- (ii) Define a random telegraph process. Show that it is a covariance stationary process. (8)
14. (a) (i) Consider two random processes  $X(t) = 3\cos(\omega t + \theta)$  and  $Y(t) = 2\cos(\omega t + \theta)$ , where  $\theta$  is a random variable uniformly distributed over  $(0, 2\pi)$ . Prove that  $R_{XY}(\tau) \leq \sqrt{R_{XX}(0)R_{YY}(0)}$ . (8)
- (ii) Find the power spectral density of a random signal with auto correlation function  $e^{-2|\tau|}$ . (8)

Or

- (b) (i) If  $X(t) = Y \cos \omega t + z \sin \omega t$  where  $Y, Z$  are two independent normal random variables with  $E(Y) = E(Z) = 0$ ,  $\text{Var}(Y) = \text{Var}(Z) = \sigma^2$  and  $W$  is a constant, prove that  $X(t)$  is a strict sense stationary process of order 2. (8)
- (ii) The power spectrum of a wide sense stationary process  $X(t)$  is given by  $S_{XX}(\omega) = \frac{1}{(1 + \omega^2)^2}$ . Find the auto correlation function. (8)
15. (a) (i) Prove that if the input to a time-invariant stable linear system is a wide sense process then the output also is a wide sense process. (8)
- (ii) A random process  $X(t)$  with  $R_{XX}(\tau) = e^{-2|\tau|}$  is the input to a linear system whose impulse response is  $h(t) = 2e^{-t}$ ,  $t > 0$ . Find the cross correlation coefficient  $R_{XY}(\tau)$  between the input process  $X(t)$  and output process  $Y(t)$ . (8)

Or

- (b) (i) Let  $X(t)$  be a wide sense stationary process which is the input to a linear time invariant system with unit impulse  $h(t)$  and output  $Y(t)$ . Prove that  $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$  where  $H(\omega)$  is the Fourier transform of  $h(t)$ . (8)
- (ii) Let  $Y(t) = X(t) + N(t)$  be a wide sense stationary process where  $X(t)$  is the actual signal and  $N(t)$  is the zero mean noise process with variance  $\sigma_N^2$ , and independent of  $X(t)$ . Find the power spectral density of  $Y(t)$ . (8)



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