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Question Paper Code : 71773

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Electronics and Communication Engineering

MA 2261/MA 45/MA 1253/080380009/10177 PR 401 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008/2010)

Time : Three hours Maximum : 100 marks

(Use of Statistical tables is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If a random variable X takes values 1, 2, 3, 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution of X.

2. Find the moment generating function of Poisson distribution.

3. The joint pdf of (X,Y) is given by $f(x,y)=k\ xye^{-(x^2+y^2)}$; $x>0,\ y>0$. Find the value of k.

4. Define the distribution function of two dimensional random variable (X,Y). State any one property.

5. Define a Markov process.

6. Prove that the sum of two independent Poisson processes is a Poisson process.

7. Define power spectral density function of a stationary random process.

8. If $R_{XX}(\tau)=\dfrac{25\tau^2+36}{6.25\tau^2+4}$. Find the mean and variance of X.

9. Define a linear system with random output.

10. State any two properties of cross power density spectrum.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random variable X has cdf $F(x)=\begin{cases}0 & \text{if } x<-1\\ a(1+x) & \text{if } -1\le x<1\\ 1 & \text{if } x\ge 1\end{cases}$. Find the value of a. Find $P(X>1/4)$ and $P(-0.5\le X\le 0)$. (8)

(ii) Obtain the moment generating function of geometric distribution. Hence, find its mean and variance. (8)

Or

(b) (i) If X is uniformly distributed with $E(X)=1$ and $\text{var}(X)=4/3$, find $P(X<0)$. (8)

(ii) Obtain the moment generating function of exponential distribution. Hence compute the first four moments. (8)

12. (a) (i) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 blue balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the probability distribution of X and Y. (8)

(ii) The random variables X and Y are related by $X-6=Y$ and $0.64X-4.08=0$. Find the mean of X and Y; and correlation coefficient between X and Y. (8)

Or

(b) (i) The random variables X and Y have joint pdf $f(x,y)=\begin{cases}x^2+\dfrac{xy}{3}; & 0<x<1,\ 0<y<2\\ 0 & \text{otherwise}\end{cases}$. Find the marginal density of X and marginal density of Y. Find the conditional density of X given Y. (8)

(ii) A random sample of size 100 is taken from a population whose mean $\mu=60$ and variance $\sigma^2=400$. Using central limit theorem with what probability can we assert that the mean of the sample will not differ from μ by more than 4. (8)

13. (a) (i) Examine whether $X(t)=A\cos\lambda t+B\sin\lambda t$ where A and B are random variables such that $E(A)=E(B)=0$; $E(A^2)=E(B^2)$; $E(AB)=0$, is wide sense stationary. (8)

(ii) Find the auto correlation function of the Poisson process. (8)

Or

(b) (i) Suppose $X(t)$ is a normal process with mean $\mu(t)=3$, $C_x(t_1,t_2)=4e^{-0.2|t_1-t_2|}$. Find $P(X(5)\le 2)$ and $P(|X(8)-X(5)|\le 1)$. (8)

(ii) Define a random telegraph process. Show that it is a covariance stationary process. (8)

14. (a) (i) Consider two random processes $X(t)=3\cos(wt+\theta)$ and $Y(t)=2\cos(wt+\theta)$, where θ is a random variable uniformly distributed over $(0,2\pi)$. Prove that $R_{XY}(\tau)\le\sqrt{R_{XX}(0)R_{YY}(0)}$. (8)

(ii) Find the power spectral density of a random signal with auto correlation function $e^{-\lambda|\tau|}$. (8)

Or

(b) (i) If $X(t)=Y\cos wt+z\sin wt$ where Y,Z are two independent normal random variables with $E(Y)=E(Z)=0$, $Var(Y)=Var(Z)=\sigma^2$ and W is a constant, prove that $X(t)$ is a strict sense stationary process of order 2. (8)

(ii) The power spectrum of a wide sense stationary process $X(t)$ is given by $S_{XX}(w)=\frac{1}{(1+w^2)^2}$. Find the auto correlation function. (8)

15. (a) (i) Prove that if the input to a time-invariant stable linear system is a wide sense process then the output also is a wide sense process. (8)

(ii) A random process $X(t)$ with $R_{XX}(\tau)=e^{-2|\tau|}$ is the input to a linear system whose impulse response is $h(t)=2e^{-t}$, $t>0$. Find the cross correlation coefficient $R_{XY}(\tau)$ between the input process $X(t)$ and output process $Y(t)$. (8)

Or

(b) (i) Let $X(t)$ be a wide sense stationary process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$. Prove that $S_{YY}(w)=|H(w)|^2 S_{XX}(w)$ where $H(w)$ is the Fourier transform of $h(t)$. (8)

(ii) Let $Y(t)=X(t)+N(t)$ be a wide sense stationary process where $X(t)$ is the actual signal and $N(t)$ is the zero mean noise process with variance σ_N^2, and independent of $X(t)$. Find the power spectral density of $Y(t)$. (8)