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Question Paper Code : 51773

B.E. /B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fourth Semester

Electronics and Communication Engineering

MA2261/MA45/MA1253/10177PR401/080380009 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulations 2008/2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. A random variable X has the probability for $P(x) = 1/2^x, x = 1, 2, \ldots$ find its m.g.f.

2. If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

find a.

3. Two random variables X and Y have the joint p.d.f.

$$F_{XY}(x, y) = \begin{cases} xy/96, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise;} \end{cases} \quad \text{find Cov(X, Y)}$$

4. From the following table find all marginal distributions :

Y\X	i = 1	i = 2	i = 3
j = 0	$\frac{1}{2}$	$\frac{1}{6}$	0
j = 1	0	$\frac{1}{9}$	$\frac{1}{5}$
j = 2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{2}{15}$

5. Define a random process and give an example.

6. Consider the random process $X(t) = \cos(w_0 t + \theta)$ with θ uniformly distributed in the interval $(-\pi, \pi)$. Check whether $x(t)$ is stationary or not.

7. Define an Auto correlation function and prove that for a WSS process $\{x(t)\}$ $R_{XX}(-\tau) = R_{XX}(\tau)$

8. Write any two properties of cross correlation.

9. Define a system and when is it called a linear system ?

10. The power spectral density of a random process $\{x(t)\}$ is given by

$$S_{xx}(w) = \begin{cases} \pi, & |w| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find its auto correlation.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) The random variable X has the p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$. Find the density function of the variable $Y = 2X + 5$ and $Y = X^2$. **(8)**

(ii) A man with n keys want to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials required to open the door, (i) if unsuccessful keys are not eliminated from further selection and (ii) if they are eliminated from further selection. **(8)**

OR

(b) (i) Show that from a uniform distribution with $f(x) = \frac{1}{2a}$, $-a < x < a$ the m.g.f. about origin is $\frac{\sin hat}{at}$. Also show that the raw moment of odd order vanish and the raw moments of even order are given by $\mu'_{2n} = \frac{a^{2n}}{2n+1}$. **(8)**

(ii) If X and Y are independent Poisson random variables, show that the conditional distribution of X given X + Y is a binomial distribution. **(8)**

12. (a) (i) If the joint distribution function of X and Y is given by

$$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$ **(10)**

(ii) Two random variables X and Y are defined with Y = 4X + 9 find the correlation coefficient between X and Y. **(6)**

OR

(b) (i) The joint p.d.f. of a two dimensional random variable is given by

$$f(x, y) = \begin{cases} xe^{-(x(y+1))}, & x \geq 0, y \geq 0, \\ 0, & \text{otherwise} \end{cases}$$

show that the regression of Y on X is not linear. **(6)**

(ii) Two independent random variables X and Y follow exponential distribution with parameter $\lambda = 1$. Find the p.d.f. of U = X – Y. **(10)**

13. (a) (i) Classify a random process with examples. **(6)**

(ii) Given a random variable Ω with density f(w) and another random variable ϕ uniformly distributed in $(-\pi, \pi)$ and independent of Ω and $X(t) = a \cos(\Omega t + \phi)$ prove that $\{x(t)\}$ is a wide sense stationary process. **(10)**

OR

(b) (i) Prove that the sum of two independent Poisson processes is a Poisson process. **(8)**

(ii) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2b|\tau|}$. Determine the power density spectrum of the random telegraph signal. **(8)**

14. (a) (i) Find the power spectral density of Binary Transmission process, where auto correlation function is

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{2} & |\tau| \leq T \\ 0 & \text{otherwise} \end{cases}$$ **(8)**

(ii) The cross power spectrum of a real random processes {X(t)} and {Y(t)} is given by

$$S_{XX}(w) = \begin{cases} a + j.bw & |w| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the cross correlation function. **(8)**

OR

(b) (i) $\{x(t)\}$ is a stationary random process with power spectral density $S_{XX}(w)$ and Y(t) is another independent random process $Y(t) = A\cos(w_0(t) + \theta)$ where θ is a random variable uniformly distributed over $(-\pi, \pi)$. Find the P.S.D of $\{Z(t)\}$ where $Z(t) = X(t).Y(t)$ **(8)**

(ii) State Wiener-Khinchine relation and define cross power spectral density and its properties. **(8)**

15. (a) (i) Given that a process $x(t)$ has the autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha|\tau|}$ where $A > 0$, $\alpha > 0$ and w_0 are real constants, find the power spectrum of $(x(t))$. **(10)**

(ii) A system has an impulse response $h(t) = e^{-\beta t}U(T)$ find the p.s.d. of the output Y(t) corresponding to the input X(t). **(6)**

OR

(b) (i) The cross power spectrum of real random processes X(t) and Y(t) is given by $S_{xy}(w) = \begin{cases} a + jbw & |\omega| < 1 \\ 0 & \text{otherwise} \end{cases}$ find the cross correlation function. **(8)**

(ii) Show that $S_{yy}(w) = |H(w)|^2 S_{xx}(w)$ where $S_{xx}(w)$ and $S_{yy}(w)$ are the power spectral density functions of the input $x(t)$ and the output y(t) and H(w) is the system transfer function. **(8)**
