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MA45 - Probability.

Question Paper Code : 11486

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Fourth Semester

Common to ECE and Biomedical Engineering

MA 2261/181403/MA 45/MA 1253/10177 PR 401/080380009 — PROBABILITY AND RANDOM PROCESSES

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — $(10 \times 2 = 20$ marks)

1. The moment generating function of a random variable X is given by $M(t) = e^{3(e^t - 1)}$. What is $P[X = 0]$?

2. An experiment succeeds twice as often as it fails. Find the chance that in the next 4 trials, there shall be at least one success.

3. Find the marginal density functions of X and Y if
$$f(x,y) = \begin{cases} \frac{6}{5}\left(x + y^2\right), & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

4. Find the acute angle between the two lines of regression, assuming the two lines of regression.

5. Define a strictly stationary random process.

6. Prove that sum of two independent Poisson processes is again a Poisson process.

7. Find the variance of the stationary process $\{X(t)\}$ whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

8. Prove that for a WSS process $\{X(t)\}, R_{XX}(t, t + \tau)$ is an even function of τ.

9. Find the system Transfer function, if a Linear Time Invariant system has an impulse function $H(t) = \begin{cases} \frac{1}{2c}; & |t| \le c \\ 0; & |t| \ge c \end{cases}$.

10. Define Band-Limited white noise.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the probability density of X is given by $f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1, \\ 0 & \text{otherwise} \end{cases}$ find its r^{th} Moment. Hence evaluate $E[(2X+1)^2]$. (6)

(ii) Find MGF corresponding to the distribution $f(\theta) = \begin{cases} \frac{1}{2}e^{-\theta/2} & \theta > 0 \\ 0 & \text{otherwise} \end{cases}$ and hence find its mean and variance. (6)

(iii) Show that for the probability function

$$p(x) = P(X = x) = \begin{cases} \frac{1}{x(x+1)}, & x = 1, 2, 3 \ldots.. \\ 0 & \text{otherwise} \end{cases}$$ $E(X)$ does not exist. (4)

Or

(b) (i) Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable X normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by

(1) at least 44.5 cc/min

(2) utmost 35.0 cc/min

(3) anywhere from 30.0 to 40.0 cc/mm. (8)

(ii) The random variable X has exponential distribution with $f(x) = f(X) = f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & \text{othewise} \end{cases}$

Find the density function of the variable given by

(1) $Y = 3X + 5$

(2) $Y = X^2$. (8)

12. (a) (i) The joint pdf of a two-dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$

Compute $P(Y < 1/2), P(X > 1 \mid Y < 1/2)$ and $P(X + Y \le 1)$. (8)

(ii) If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient between $(X + Y)$ and $(X - Y)$. (8)

Or

(b) If X and Y are independent random variables with probability density functions $f_X(x) = 4e^{-4x}, x \ge 0; f_Y(y) = 2e^{-2y}, y \ge 0$ respectively.

(i) Find the density function of $U = \frac{X}{X+Y}, V = X + Y$. (11)

(ii) Are U and V independent? (2)

(iii) What is $P(U > 0.5)$? (3)

13. (a) (i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2.. \\ \frac{at}{1+at}, & n = 0 \end{cases}$$

Find the mean and variance of the process. Is the process first-order stationary? (8)

(ii) If the WSS process $\{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$, where θ is uniformly distributed over $(-\pi, \pi)$, prove that $\{X(t)\}$ is correlation ergodic. (8)

Or

(b) (i) If the process $\{X(t); t \ge 0\}$ is a Poisson process with parameter λ, obtain $P[X(t) = n]$. Is the process first order stationary? (8)

(ii) Prove that a random telegraph signal process $Y(t) = \alpha X(t)$ is a Wide Sense Stationary Process when α is a random variable which is independent of $X(t)$, assumes values -1 and $+1$ with equal probability and $R_{XX}(t_1, t_2) = e^{-2\lambda|t_1 - t_2|}$. (8)

14. (a) (i) If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively then prove that $|R_{XX}(\tau)| \leq \sqrt{R_{XX}(0) R_{YY}(0)}$. Establish any two properties of auto correlation function $R_{XX}(\tau)$. (8)

(ii) Find the power spectral density of the random process whose auto correlation function is $R(\tau) = \begin{cases} 1 - |\tau|, & \text{for } |\tau| \leq 1 \\ 0, & \text{elsewhere} \end{cases}$. (8)

Or

(b) State and prove Wiener Khintchine theorem and hence find the power spectral density of a WSS process $X(t)$ which has an autocorrelation $R_{xx}(\tau) = A_0 [1 - |\tau|/T], -T \leq t \leq T$. (16)

15. (a) (i) Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output $\{Y(t)\}$ is a WSS process. Also find $R_{XY}(\tau)$. (8)

(ii) If $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2|\tau|}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \dfrac{1}{\omega + 2i}$. (8)

Or

(b) (i) If $Y(t) = A\cos(w_0 t + \theta) + N(t)$, where A is a constant, θ is a random variable with a uniform distribution in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with power spectral density $S_{NN}(w) = \begin{cases} \dfrac{N_0}{2}, & \text{for } |w - w_0| < w_B \\ 0, & \text{elsewhere} \end{cases}$. Find the power spectral density $\{Y(t)\}$. Assume that $\{N(t)\}$ and θ are independent. (10)

(ii) A system has an impulse response $h(t) = e^{-\beta t} U(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$. (6)

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