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Question Paper Code : T3048

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009

Third Semester

Civil Engineering

MA 2211 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 Marks

Answer ALL Questions

PART A — (10 × 2 = 20 Marks)

1. State the sufficient condition for a function $f(x)$ to be expressed as a Fourier series.
2. Obtain the first term of the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.
3. Find the Fourier transform of
$$f(x) = \begin{cases} e^{ikx}, & a < x < b \\ 0 & x < a \text{ and } x > b. \end{cases}$$
4. Find the Fourier sine transform of $\frac{1}{x}$.
5. Find the partial differential equation of all planes cutting equal intercepts from the x and y axes.
6. Solve $(D^3 - 2D^2D')z = 0$.
7. Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.
8. Write down all possible solutions of one dimensional wave equation.
9. If $F(z) = \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)\left(z-\frac{3}{4}\right)}$, find $f(0)$.
10. Find the Z-transform of
$$x(n) = \begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

PART B — ($5 \times 16 = 80$ Marks)

11. (a) (i) Obtain the Fourier series of the periodic function defined by

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \infty = \frac{\pi^2}{8}$. (10)

(ii) Compute upto first harmonics of the Fourier series of $f(x)$ given by the following table

x :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
$f(x)$:	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

(6)

Or

(b) (i) Expand $f(x) = x - x^2$ as a Fourier series in $-L < x < L$ and using this series find the root mean square value of $f(x)$ in the interval. (10)

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 < x < 1$. (6)

12. (a) (i) Find the Fourier transform of $f(x)\begin{cases} = 1 - |x| & \text{if } |x| < 1 \\ = 0 & \text{if } |x| \geq 1 \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin^4 t}{t^4}\, dt$. (8)

(ii) Evaluate $\int_0^\infty \frac{dx}{(4 + x^2)(25 + x^2)}$ using transform methods. (8)

Or

(b) (i) Find the Fourier cosine transform of e^{-x^2}. (8)

(ii) Prove that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine and cosine transforms. (8)