

V199

B.E./B.TECH DEGREE EXAMINATIONS, NOV/DEC-2011

REGULATIONS 2008

THIRD SEMESTER

**MA31 – TRANSFORMS AND PARTIAL DIFFERENTIAL
EQUATIONS**

CIVIL ENGINEERING

(Common to all B.E/B.Tech)

Time: Three Hours

Maximum: 100 marks

ANSWER ALL QUESTIONS

PART-A (10×2=20 marks)

1. Find the complete integral of $p + q = pq$.
2. Find the particular integral of $(D^2 + 6DD' + 9D'^2)z = e^{2x}$.
3. Write the Fourier sine series of $f(x)$ in $(0,2)$.
4. Find the R.M.S value of $f(x) = x^2$ in $(-\pi, \pi)$.
5. Write the Fourier transform pair.
6. If $F(s)$ is the Fourier transform of $f(x)$, find the Fourier transform of $f(x - a)$.
7. Find $L^{-1}[1]$.
8. State the convolution theorem for Laplace transforms.
9. What is the region of convergence in Z-transform?

10. Find the Z-transform of an .

PART-B (5×16=80 marks)

11. (a) (i) Solve: $(3z - 4y)p + (4x - 2z)q = 2y - 3x$. (8)

(ii) Solve: (8)

$$(D^2 + DD' - 6D'^2)z = \cos(2x + y) + e^{x-y}.$$

Or

(b) (i) Solve: $z = px + qy - 2\sqrt{pq}$. (8)

(ii) Solve: $(D^2 + 4DD' - 5D'^2)z = y^2 + x$. (8)

12. (a) Find the Fourier series of $f(x) = x^2$ (16)
in $-\pi \leq x \leq \pi$ and hence find

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

Or

(b) Find the Fourier cosine series of $f(x) = x \sin x$ (16)
in $0 < x < \pi$ and hence deduce the value

of $1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} - \dots$

13. (a) Find the Fourier transform of $f(x)$ if (16)

$$f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a > 0 \end{cases} \text{. Hence find}$$

$$\int_0^{\infty} \frac{\sin^2 t}{t^2} dt \text{ and } \int_0^{\infty} \frac{\sin^4 t}{t^4} dt.$$

Or

- (b) (i) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(y^2 + b^2)}$ using (8)
transform method.
(ii) Find the Fourier cosine transforms of (8)
 $e^{-a^2 x^2}$

14. (a) (i) Find the inverse Laplace transform of (8)

$$\frac{e^{-s}}{(s-2)(s+3)}$$

- (ii) Find the Laplace transform of a periodic (8)
function $f(t) = \begin{cases} 1 & \text{in } 0 < t < a \\ -1 & \text{in } a < t < 2a \end{cases}$

Or

- (b) (i) Use convolution theorem to evaluate (8)

$$\int_0^t \sin u \cos(t-u) du$$

by Laplace Transform

- Technique
(ii) Find $L(t^2 e^{-t} \cos t)$ (8)

15. (a) (i) Using convolution theorem, find (8)

$$Z^{-1}\left(\frac{z^2}{(z-1)(z-2)}\right).$$

(ii) Find the Z-transforms of (8)

$$\frac{1}{(n+1)} \text{ and } \frac{1}{n+i!}.$$

Or

(b) (i) Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given (8)

that $y_0 = 0, y_1 = 0$.

(ii) State and prove second shifting theorem (8)
of Z-transform