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B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Third Semester

Civil Engineering

MA 2211/MA 31/MA 1201 A/CK 201/10177 MA 301/080100008/080210001/ MAU 211/ETMA 9211 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulation 2008)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the co-efficient b_n of the Fourier series for the function $f(x) = x \sin x$ in $(-2, 2)$.

2. Define Root Mean Square value of a function $f(x)$ over the interval (a, b).

3. Find the Fourier transform of $e^{-\alpha|x|}$, $\alpha > 0$.

4. State convolution theorem in Fourier transform.

5. Eliminate the arbitrary function 'f' from $z = f\left(\dfrac{y}{x}\right)$ and form the PDE.

6. Solve : $(D-1)(D-D'+1)z = 0$.

7. An insulated rod of length 60 cm has its ends at A and B maintained at 20°C and 80°C respectively. Find the steady state solution of the rod.

8. A plate is bounded by the lines $x = 0$, $y = 0$, $x = l$ and $y = l$. Its faces are insulated. The edge coinciding with x-axis is kept at 100°C. The edge coinciding with y-axis is kept at 50°C. The other two edges are kept at 0°C. Write the boundary conditions that are needed for solving two dimensional heat flow equation.

9. Find the Z-transform of a^n.

10. Solve $y_{n+1} - 2y_n = 0$, given that $y(0) = 2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Fourier series expansion of $f(x) = x + x^2$ in $(-\pi, \pi)$. (8)

(ii) Find the Fourier series expansion of $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \end{cases}$. Also deduce $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \cdots \text{ to } \infty = \dfrac{\pi^2}{8}$. (8)

Or

(b) (i) Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$. (8)

(ii) Find the Fourier series as far as the second harmonic to represent the function $f(x)$ with period 6, given in the following table : (8)

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

12. (a) (i) Find the Fourier transform of $f(x)=\begin{cases}1-|x| & \text{if } |x|<1\\ 0 & \text{if } |x|>1\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^4 t}{t^4}\,dt$. (8)

(ii) Find the Fourier transform of $f(x)=\begin{cases}a^2-x^2, & |x|\le a\\ 0, & |x|>a>0\end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}\,dt=\frac{\pi}{4}$. (8)

Or

(b) (i) Find the Fourier cosine and sine transforms of $f(x)=e^{-ax}$, $a>0$ and hence deduce the inversion formula. (8)

(ii) Find the Fourier cosine transform of $e^{-a^2x^2}$, $a>0$. Hence show that the function $e^{-x^2/2}$ is self-reciprocal. (8)

13. (a) (i) Find the singular integral of $z=px+qy+p^2+pq+q^2$. (8)

(ii) Solve the partial differential equation $(x-2z)p+(2z-y)q=y-x$. (8)

Or

(b) (i) Solve : $(D^2+3DD'-4D'^2)z=\cos(2x+y)+xy$. (8)

(ii) Solve : $(D^2-DD'+2D)z=e^{2x+y}+4$. (8)

14. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position. Find the expression for the displacement at any time t. (16)

Or

(b) Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surfaces and satisfying the boundary conditions :
$u(0,y)=u(a,y)=0$, for $0\le y\le b$;
$u(x,b)=0$ and $u(x,0)=x(a-x)$, for $0\le x\le a$. (16)

15. (a) (i) Find the Z-transforms of $\sin^2\left(\frac{n\pi}{4}\right)$ and $\cos\left(\frac{n\pi}{2}+\frac{n\pi}{4}\right)$. (8)

(ii) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z+a)^2}$. (8)

Or

(b) (i) Solve the difference equation using Z-transform
$y_{(n+3)}-3y_{(n+1)}+2y_{(n)}=0$ given that $y_0=4$, $y_1=0$, $y_2=8$. (8)

(ii) Solve $y_{(n+2)}+6y_{(n+1)}+9y_{(n)}=2^n$ given that $y_0=y_1=0$. (8)
