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Question Paper Code : 71861

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Sixth Semester

Mechanical Engineering

ME 2353/ME 63/10122 ME 605 — FINITE ELEMENT ANALYSIS

(Common to Automobile Engineering, Mechanical and Automation Engineering, Industrial Engineering and Management)

(Regulation 2008/2010)

Time : Three hours | Maximum : 100 marks

(Any missing data may be suitably assumed)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Distinguish between Error in solution and Residual.

2. What are the advantages of weak formulation?

3. Derive the shape functions for a 1 D quadratic bar element.

4. What are the properties of the Stiffness matrix?

5. Write down the shape functions associated with the three noded linear triangular element and plot the variation of the same.

6. Give at least one example each for plane stress and plane strain analysis.

7. Derive the mass matrix for a 1 D linear bar element.

8. Write down the Governing equation and for 1 D longitudinal vibration of a bar fixed at one end and give the boundary conditions.

9. Derive the convection matrix for a 1 D linear bar element.

10. Write down the conduction matrix for a three noded linear triangular element.

PART B — (5 × 16 = 80 marks)

11. (a) Determine using any Weighted Residual technique the temperature distribution along a circular fin of length 6 cm and radius 1 cm. The fin is attached to a boiler whose wall temperature is 140°C and the free end is insulated. Assume convection coefficient h=10 W/cm² °C. Conduction coefficient K = 70 W/cm² °C and $T_\infty = 40°$ C. The Governing Equation for the heat transfer through the fin is given by

$$-\frac{d}{dx}\left[KA(x)\frac{dT}{dx}\right] + hp(x)(T - T_\infty) = 0$$

Assume appropriate boundary conditions and calculate the temperatures at every 1 cm from the left end.

Or

(b) Derive the governing equation for a tapered rod fixed at one end and subjected to its own self weight and a force P at the other end as shown in Fig.11(b). Let the length of the bar be l and let the cross section vary linearly from A1 at the top fixed end to A2 at the free end. E and γ represent the Young's modulus and specific weight of the material of the bar. Convert this equation into its weak form and hence determine the matrices for solving using the Ritz technique.

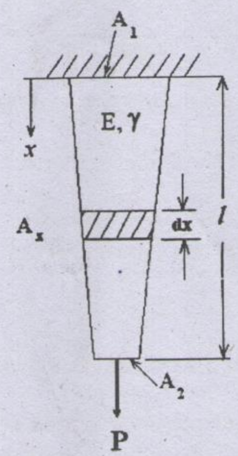

Fig.11(b)

12. (a) Determine the maximum deflection and slope in the beam, loaded as shown in Fig.12(a). Determine also the reactions at the supports. $E = 200$ GPa, $I = 20 \times 10^{-6}$ m^4 $q = 5$ kN/m and $L = 1$m

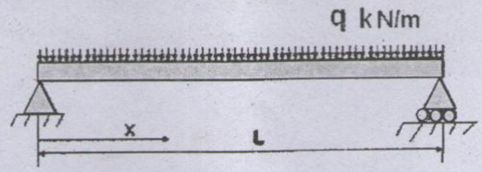

Fig.12(a)

Or

(b) Derive using Lagrangian Polynomials the shape functions for a one dimensional three noded bar element. Plot the variation of the same. Hence derive the stiffness matrix and load vector.

13. (a) (i) A bilinear rectangular element has coordinates as shown in Fig.13(a) and the nodal temperatures are $T_1 = 100°$ C, $T_2 = 60°$ C, $T_3 = 50°$ C, $T_4 = 90°$ C.

Compute the temperature at the point whose coordinates are (2.5, 2.5). Also determine the 80°C isotherm. (10)

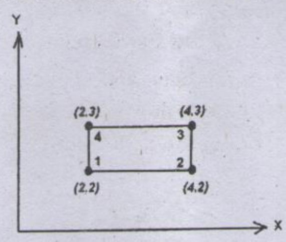

Fig.13(a)

(ii) Using Gauss Quadrature evaluate the following integral $I = \int_{-1}^{+1}\int_{-1}^{1} \frac{3+\xi^2}{2+\eta^2} d\xi d\eta$. (6)

Or

(b) (i) For the four noded element shown in Fig 13.(b)(i) determine the Jacobian and evaluate its value at the point (1/2, 1/3). (8)

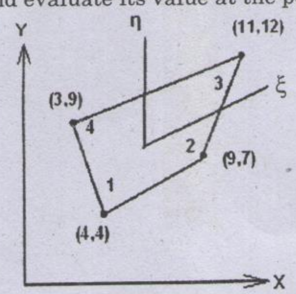

Fig.13(b)(i)

(ii) A plate of dimensions 15 cm × 6 cm × 1 cm is subjected to an axial pull of 5 kN.

Assuming a typical element is of dimensions as shown in the fig.13(b)(ii). Determine the strain displacement matrix and constitutive matrix. E = 200 GPa, $\mu = 0.3$, $t = 10\ mm$. (8)

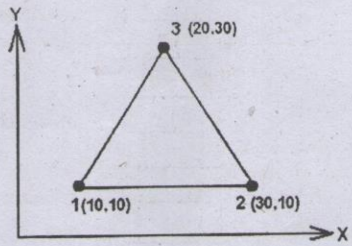

Fig.13(b)(ii)

14. (a) **Determine the first two natural frequencies of transverse vibration of the cantilever beam shown in Fig.14(a) and plot the mode shapes.**

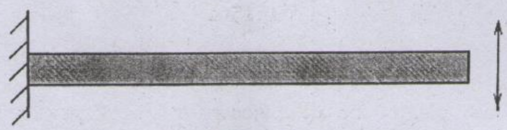

Fig.14(a)

Or

(b) **Determine the first two natural frequencies of longitudinal vibration of the bar shown in Fig.14(b) assuming that the bar is discretised into two elements as shown. E and ρ represent the Youngs Modulus and mass density of the material of the bar.**

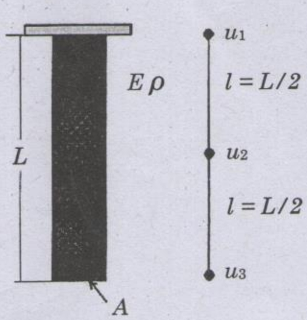

Fig.14(b)

15. (a) A composite wall consists of three materials as shown in Fig.15(a). The inside wall temperature is 200°C and the outside air temperature is 50°C with a convection coefficient of 10 W/cm² °C. Determine the temperature along the composite wall.

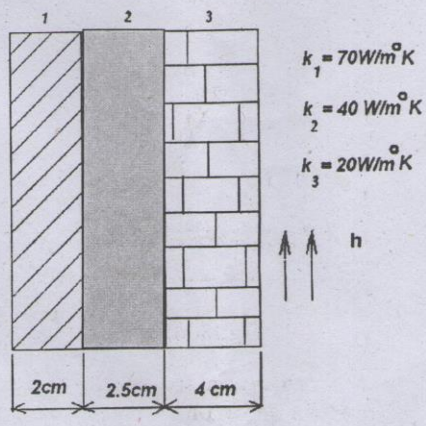

Fig.15(a)

Or

(b) A two dimensional fin is subjected to heat transfer by conduction and convection. It is discretised as shown in Fig.15(b), into two elements using linear triangular elements. Derive the conduction, and thermal load vector. How is convection accounted for in solving the problem using Finite element method?

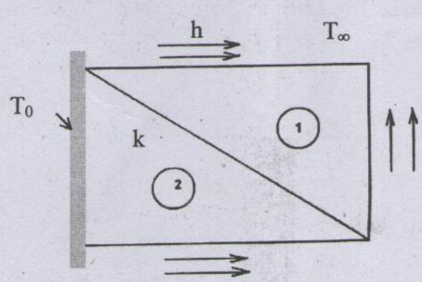

Fig.15(b)

$$\text{Stiffness Matrix } [K]^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\{f\}^e = \frac{ql}{2}\begin{Bmatrix} 1 \\ l/6 \\ 1 \\ -l/6 \end{Bmatrix}$$

$$[M] = \frac{\rho A\ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix}$$

No. of points	Location	Weight W_i
1	$\xi_1 = 0.00000$	2,00000
2	$\xi_1, \xi_2 \pm 0.57735$	1.000000
3	$\xi_1, \xi_3 \pm 0.77459$	0.55555
	$\xi_2 = 0.00000$	0.00000