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Question Paper Code: E3121

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2010

Third Semester

Civil Engineering

MA2211 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all Branches of B.E./B.Tech)

(Regulation 2008)

Time: Three hours — Maximum: 100 marks

Answer ALL Questions

PART A — (10 × 2 = 20 Marks)

1. Write the conditions for a function $f(x)$ to satisfy for the existence of a Fourier series.

2. If $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$, deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$.

3. Find the Fourier cosine transform of e^{-ax}, $x \geq 0$.

4. If $F(s)$ is the Fourier transform of $f(x)$, show that $F(f(x-a)) = e^{ias}F(s)$.

5. Form the partial differential equation by eliminating the constants a and b from $z = (x^2+a^2)(y^2+b^2)$.

6. Solve the partial differential equation $pq = x$.

7. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest in this position, write the boundary conditions.

8. Write all three possible solutions of steady state two-dimensional heat equation.

9. Find the Z-transform of $\sin\frac{n\pi}{2}$.

10. Find the difference equation generated by $y_n = a_n + b2^n$.

PART B — (5 × 16 = 80 Marks)

11. (a) (i) Find the Fourier series for $f(x) = 2x - x^2$ in the interval $0 < x < 2$. (8)

(ii) Find the half range cosine series of the function $f(x) = x(\pi - x)$ in the interval $0 < x < \pi$. Hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots = \frac{\pi^4}{90}$. (8)

Or

(b) (i) Find the complex form of the Fourier series of $f(x) = e^{ax}$, $-\pi < x < \pi$. (8)

(ii) Find the first two harmonics of the Fourier series from the following table: (8)

x:	0	1	2	3	4	5
y:	9	18	24	28	26	20

12. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases}$. Hence deduce that the value of $\int_0^\infty \frac{\sin^4 t}{t^4}\, dt$. (10)

(ii) Show that the Fourier transform of $e^{\frac{-x^2}{2}}$ is $e^{\frac{-s^2}{2}}$. (6)

Or

(b) (i) Find the Fourier sine and cosine transforms of

$$f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases}.$$ (8)

(ii) Using Fourier cosine transform method, evaluate $\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)}$. (8)

13. (a) Solve :

(i) $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$ (8)

(ii) $p(1+q) = qz$ (4)

(iii) $p^2 + q^2 = x^2 + y^2$. (4)

Or

(b) (i) Find the partial differential equation of all planes which are at a constant distance 'a' from the origin. (8)

(ii) Solve $\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right)z = \sin(x+2y)$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. (8)

14. (a) A tightly stretched string of length 'l' has its ends fastened at $x = 0$ and $x = l$. The mid-point of the string is then taken to height 'b' and released from rest in that position. Find the lateral displacement of a point of the string at time 't' from the instant of release. (16)

Or

(b) A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that may be considered infinite in length without introducing appreciable error. The temperature at short edge $y = 0$ is given by $u = \begin{cases} 20x & \text{for } 0 \le x \le 5 \\ 20(10-x) & \text{for } 5 \le x \le 10 \end{cases}$ and the other three edges are kept at 0°C. Find the steady state temperature at any point in the plate. (16)

15. (a) (i) Solve by Z-transform $u_{n+2} - 2u_{n+1} + u_n = 2^n$ with $u_0 = 2$ and $u_1 = 1$. (8)

(ii) Using convolution theorem, find the inverse Z-transform of $\left(\frac{z}{z-4}\right)^3$. (8)

Or

(b) (i) Find $Z^{-1}\left[\frac{z\left(z^2 - z + 2\right)}{(z+1)(z-1)^2}\right]$ and $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$. (6 + 4)

(ii) Find $Z\left(na^n \sin n\theta\right)$. (6)
