NUMERICAL METHODS

1. Lagrange's Interpolation:

$$Y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

2. Lagrange's Inverse Interpolation:

$$X = \frac{(y-y_1)(y-y_2).....(y-y_n)}{(y_0-y_1)(y_0-y_2).....(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)...(y-y_n)}{(y_1-y_0)(y_1-y_2)...(y_1-y_n)}x_1 + \cdots +$$

$$\frac{(y-y_0)(y-y_1)(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_{n-1})}x_n$$

3. Newton's Divided Difference Interpolations:

$$f(x) = f(x_0) + (x - x_0)\, f\left(x_{0,}\, x_1\right) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + \cdots$$

$$+ (x - x_0)(x - x_1) \ldots (x - x_{n-1})\, f(x_0, x_1, \ldots x_n)$$

4. Newton's forward interpolation formula

$$P_n(x) = P_n\,(x_0 + uh) = E^u P_n(x_0) = E^u y_0$$

$$= (1 + \Delta)^u y_0$$

$$= y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

Where $u = \frac{x-x_0}{h}$

5. Newton's backward difference formula

$$P_n(x) = P_n(x_n + vh) = E^v P_n(x_n)$$

$= (1-\nabla)^{-v} y_n$ Since $E = (1-\nabla)^{-1}$

$$= \left[1 + v\nabla + \frac{v(v+1)}{2!}\nabla^2 + \frac{v(v+1)(v+2)}{3!}\nabla^3 + \cdots\right] y_n$$

$$= y_n + v\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

Where $v = \frac{x-x_n}{n}$

6. Newton's Backward Difference Formula for the first and Second Order Derivatives:

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_n$$

Where $v = \frac{x-x_n}{h}$

Here, $x = x_n$

$y(x) = y_n$

$y'(x) = y'_n = \lim_{h\to 0}\frac{y(x+h)-y(x)}{h}$

$y''(x) = y''_n$

7. Newton's Backward Difference Formula for the First and Second Order

Derivatives:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \cdots\right]$$

8. Numerical Single Integration By:

Trapezoidal Rule:

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_1) + (y_1 + y_2) + \cdots + (y_{n-1} + y_n)]$$

$$= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

9. Simpson's $^1/_{3^{rd}}$ Rule:

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[(y_0 + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \cdots$$

$$+ (y_{n-2} + 4y_{n-1} + y_n)]$$

$$= \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1})$$

$$+ 2(y_2 + y_4 + \cdots + y_{n-2})$$

10. Simpson's Three-Eight Rule:

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}[y_0 + 3y_1 + 3y_2 + y_3) + (y_3 + 3y_4 + 3y_5 + y_6) + \cdots$$

$$+ (y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)]$$

$$= \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) +$$

$$2(y_3 + y_6 + \cdots + y_{n-3})]$$

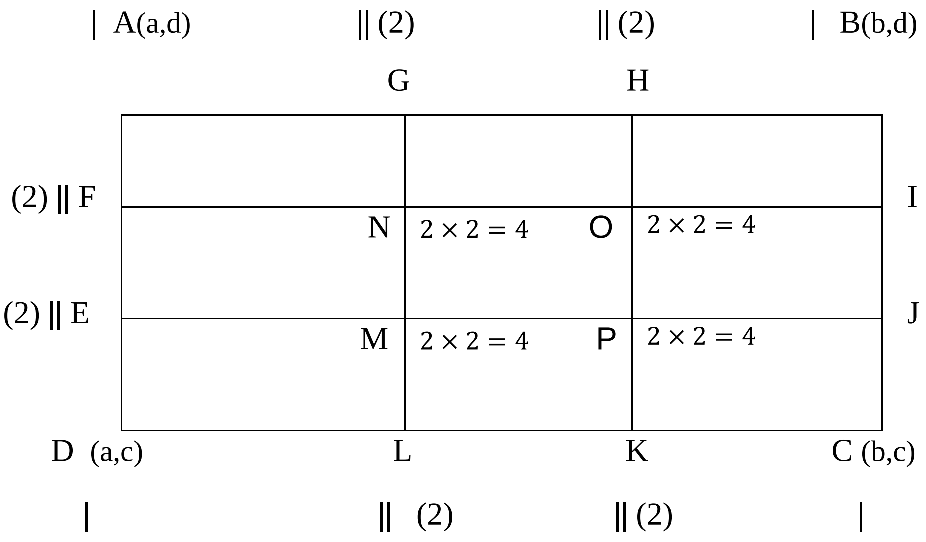
11. Numerical Double Integrals:

Trapezoidal Rule for Double Integration:

$$I = \int_{y_j}^{y_j+1} \frac{h}{2}[f(x_{i,}y) + f(x_{i+1}, y)]dy$$

$$= \frac{h}{2}\left[\int_{y_j}^{y_j+1} f(x_i\,, y)\, dy + \int_{y_j}^{y_j+1} f(x_{i+1}\,, y) dy\right]$$

12. Extension to General form of trapezoidal Rule:



$$I = \int_c^d \int_a^b f(x,y)dx\,dy$$

$$= \left(\frac{h}{2}\right)\left(\frac{h}{2}\right)[(A+B+C+D)$$

$$+2(E+F+G+H+I+j+K+L)$$

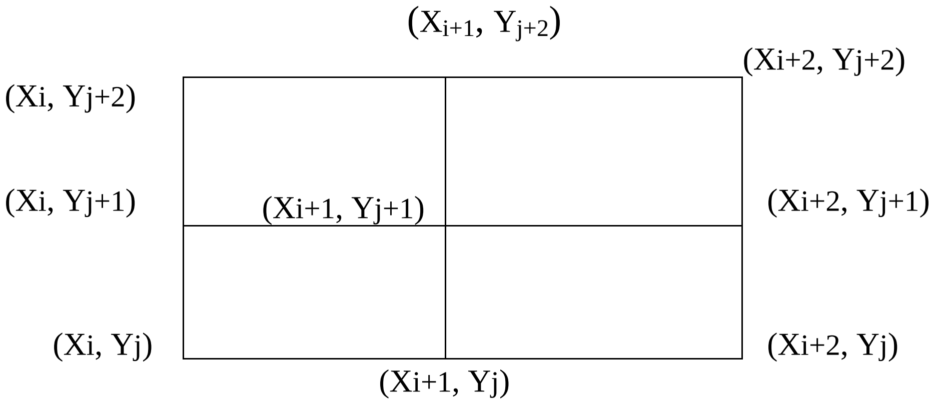
$$+4(M+N+O+P)]$$

$I = \frac{hk}{4}[(sum\ of\ values\ of\ f\ at\ four\ corners) +$

$2(Sum\ of\ the\ values\ of\ f\ at\ the\ remaining\ nodes\ on\ the\ boundary) +$

$4(Sum\ of\ the\ values\ of\ the\ values\ of\ f\ at\ the\ interior\ nodes)]$

13. Simpson's Rule for Double Integration:



$$I = \frac{hk}{9}[(f_{i,j} + f_{i,j+2} + f_{i+2,j} + f_{i+2,j+2})$$

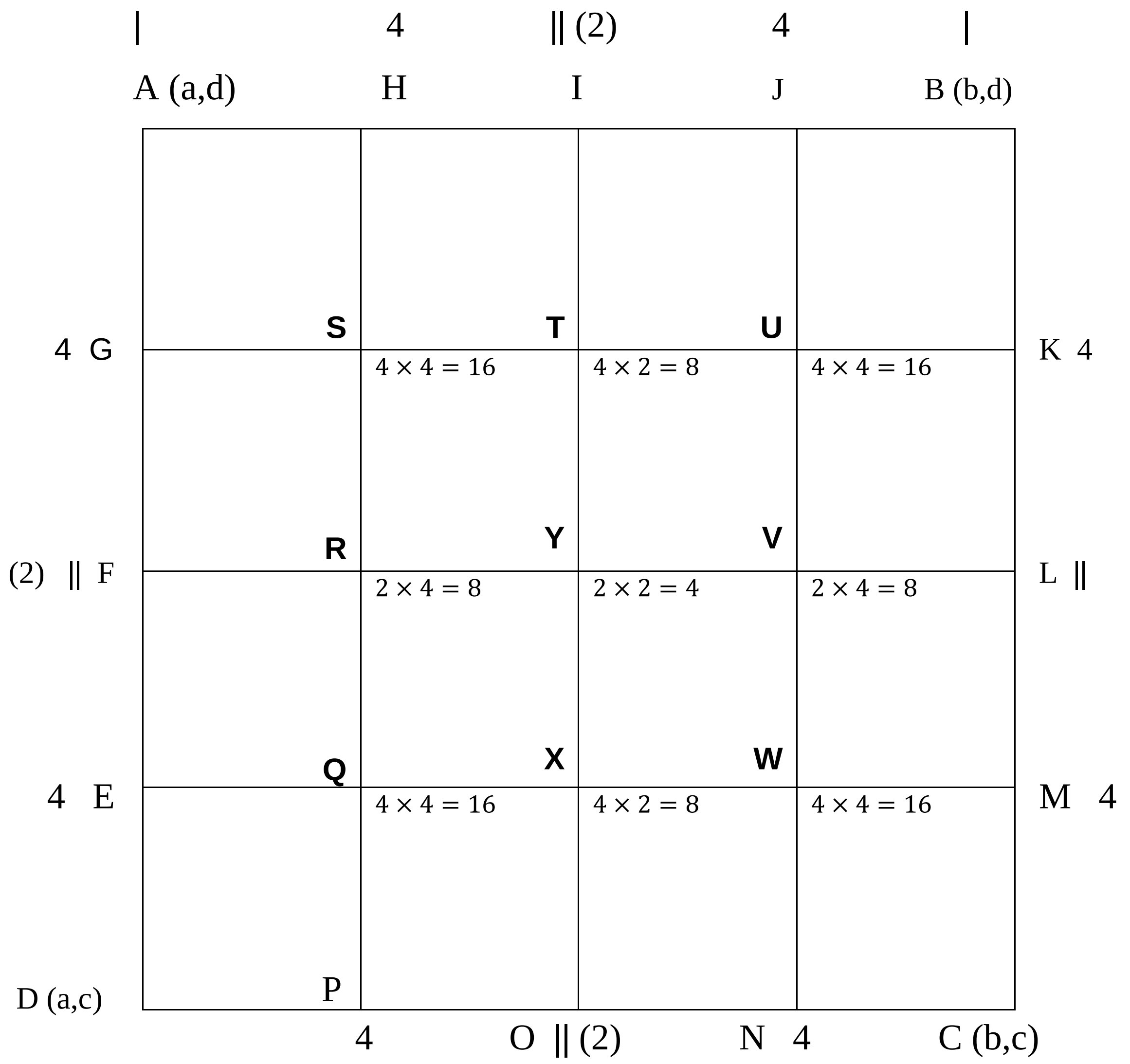
$$+4(f_{i,j+1} + f_{i+1,j} + f_{i+1,j+2} + f_{i+2,j+1}) + 16f_{i+1,j+1}]$$

$I = \frac{hk}{9}[(Sum\ of\ the\ values\ of\ f\ at\ the\ 4\ corners) +$

$4\ (sum\ of\ the\ values\ of\ f\ at\ the\ remaining\ nodes\ in\ the\ boundary) +$

$16\ (Value\ of\ f\ at\ the\ central\ point)]$

14. Extension to General form of Simpson's Rule:



$I = \frac{hk}{9}[(Sum\ of\ the\ values\ of\ f\ at\ the\ four\ corners)\ \ +$

$2(Sum\ of\ the\ values\ of\ f\ at\ the\ odd\ positions\ on\ the\ bundary\ except\ the\ \ corners) +$

$4(Sum\ of\ the\ values\ of\ f\ at\ the\ even\ positions\ on\ the\ boundary) +$

$\{4(Sum\ of\ the\ values\ of\ f\ at\ odd\ positions) +$

$8(Sum\ of\ the\ values\ of\ f at\ even\ positions) on\ the\ odd\ rows\ of\ the\ matrix\ except\ boundary\ rows\} +$

$\{8(Sum\ of\ the\ values\ of\ f\ at\ the\ odd\ positions)\ \ +$

$16(sum\ of\ the\ values\ of\ f\ at\ the\ evenpositions) on\ the\ evenrows\ of\ the\ matrix\}]$

15. Euler and Modified Euler Method:

$$y_1 = y_0 + (x_1 - x_0) f(x_{0,} y_0)$$

$$y_1 = y_0 + hf(x_{0,} y_0)$$

$$y_2 = y_1 + hf(x_{1,} y_1)$$

$$y_{n+1} = y_n + hf\left(x_{n,} y_n\right), n = 0, 1, 2, ..$$

16. Modified Euler's Method:

$$y_{n+1} = y_n + h[f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hf\left(x_{n,} y_n\right)]$$

[OR]

$$y(x + h) = y(x) + h[f\left(x + \frac{1}{2}h, y + \frac{1}{2}hf\ (x, y)\right)]$$

17. Runge-Kutta Method:

Second Order R-K Method:

$$k_1 = hf\ (x, y)$$

$$k_2 = hf[x + \frac{h}{2}, y + \frac{k_1}{2}]$$

$$\Delta y = k_2 \quad Where\ \ h = \Delta x$$

18. Third Order R-K Method:

$$k_1 = hf(x, y)$$

$$k_2 = hf[x + \frac{h}{2}, y + \frac{k_1}{2}]$$

$$k_3 = hf[x + h, y + 2k_2 - k_1]$$

$$and\ \Delta y = \frac{1}{6}(k_1 + 4k_2 + k_3)$$

19. Fourth Order R-K Method:

$$k_1 = hf(x, y)$$

$$k_2 = hf[x + \frac{h}{2}, y + \frac{k_1}{2}]$$

$$k_3 = hf[x + \frac{h}{2}, y + \frac{k_2}{2}]$$

$$k_4 = hf[x + h, y + k_3]$$

$$and\ \Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$y(x + h) = y(x) + \Delta y$$

20. Milne's Predictor and Corrector Methods for Solving First Order Equations:

$$y_{n+1,}\, p = y_{n-3} + \frac{4h}{3}[2y'_{n-2} - y'_{n-1} + 2y'_{n}]$$

$$y_{n+1,}c = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_{n} + y'_{n+1}]$$

21. Some standard Forms of the Binomial Expansion For all Values of N,

when [X]<1, We Have:

1. $(1 + x)^n = 1 + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \cdots$
2. $(1 - x)^n = 1 - \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 - \cdots$