PARTIAL FUNCTIONS AND INFINITE SERIES

To resolve in to partial fractions, degree of numerator should be less than the degree of denominator.

$$[deg. of\ Nr. < deg. of. Dr]$$

Types

1. $\frac{1}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$
2. $\frac{1}{(x+a)(x+b)(x+c)} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)}$
3. $\frac{1}{(x+a)(x+b)(x+c)(x+d)} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)} + \frac{D}{(x+d)}$
4. $\frac{ax}{(x+a)^2(x+b)} = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{x+b}$
5. $\frac{ax^2+bx+c}{(x+a)(x+b)^3} = \frac{A}{x+a} + \frac{B}{x+b} + \frac{C}{(x+b)^2} + \frac{D}{(x+b)^3}$
6. $\frac{x^3+c}{(x^2+a)(x+b)} = \frac{Ax+B}{x^2+a} + \frac{C}{x+b}$
7. $\frac{x^2\ (or)\ constant}{(x^2+a)(x^2+b)} = \frac{A}{(x^2+a)} + \frac{B}{(x^2+b)}$
8. $\frac{x^3 (or)\ x}{(x^2+a)(x^2+b)} = \frac{Ax+B}{x^2+a} + \frac{Cx+D}{x^2+b}$
9. $\frac{x^3+c}{(x+a)(x+b)^2} = A + \frac{B}{x+a} + \frac{C}{x+b} + \frac{D}{(x+b)^2}$
10. $\frac{ax^3+bx^2+cx+d}{(x+a)(x+b)} = Ax + B + \frac{C}{x+a} + \frac{D}{x+b}$

BINOMIAL SERIES:

If $|x| \leq 1\ (ie) if -1 \leq x \leq 1$, we have

1. $(1+x)^n = 1 + \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + x^n$

   $= nC_0{}_{-}nC_1x + nC_2x^2 + \cdots + nC_nx_n$

2. $(1-x)^n = 1 - \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + (-1)^n x^n$

   $= nC_0 - nC_1x + nC_2x^2 - \cdots + (-1)^n x^n$

3. $(1+x)^{-n} = 1 - \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \cdots\infty$

4. $(1-x)^{-n} = 1 + \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \cdots\infty$

From this we have

- $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots\infty$
- $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots\infty$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots\infty$
- $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots\infty$
- $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots\infty = \frac{1}{2}(1.2 + 2.3x + 3.4x^2 + \cdots)$
- $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \cdots\infty = \frac{1}{2}(1.2 - 2.3x + 3.4x^2 - \cdots)$

EXPONENTIAL SERIES:

1. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\infty$
2. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\infty$
3. $\sinh x = \frac{e^x - e^{-x}}{2} = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\infty$
4. $\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\infty$

LOGARITHMIC SERIES:

1. $log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\infty$
2. $-log_e(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\infty$
3. $\frac{1}{2}log_e\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\infty$

GEOMETRIC SERIES

1. $1 + z + z^2 + z^3 + \cdots\infty = \frac{1}{1-z}$
2. $1 - z + z^2 - z^3 + \cdots\infty = \frac{1}{1+z}$
3. $1 + z + z^2 + z^3 + \cdots z^{n-1} = \frac{z^n - 1}{z-1}$

GREGORY SERIES

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots\infty$$

Indeterminate forms

1. $\frac{0}{0}$
2. $\frac{\infty}{\infty}$
3. $0 \times \infty$
4. $\infty - \infty$
5. 1^∞
6. ∞^0
7. 0^0

Credits

POPULAR MATHS ACADEMY, NAGERCOIL

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I. L' Hopital rule for $\frac{0}{0}$ (or) $\frac{\infty}{\infty}$ form

If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$

then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Logarithmic rules

1. $log_a 1 = 0$
2. $log_a a = 1$
3. $log_b a = \frac{1}{log_a b}$
4. $log_b a . log_c b = log_c a$
5. $e^{log_e x} = x$
6. $log_a 0 = -\infty (a > 1)$
7. $\log(mn) = logm + logn$
8. $\log\left(\frac{m}{n}\right) = logm - logn$
9. $\log(m^n) = n\ logm$